Relativistic Quantum States of an Electron with Anomalous Magnetic Moment in an Electromagnetic Wave Field and a Homogeneous Magnetic Field [*]

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Abstract

The exact solution of the Dirac equation and the spectrum of electron quasi-energies in a superposition of the field of a circularly polarized electromagnetic wave and a homogeneous magnetic field parallel to the direction of wave propagation, are found taking into account the anomalous magnetic moment. It is found that taking account of the anomalous magnetic moment removes the spin degeneracy and that for intense fields the levels change radically. The shift of the radiation frequency due to the intensity of the wave field is found. This shift can be considerable.

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Contents

1 Introduction

The exact solutions of the Dirac equation in a superposition of a homogeneous magnetic field $\vec{\mathcal{B}}$ and a classical monochromatic electromagnetic wave field propagating along $\vec{\mathcal{B}}$ (the so-called Redmond configuration) were found for the first time in [1]. In [2] a transformation was introduced, in analogy with the Volkov solution [3], which reduces the solution of the Dirac equation for fields of the Redmond configuration to a product of the solution of a Klein-Gordon equation for these fields and a bispinor of a free particle. Wave functions of this form facilitate the physical interpretation, calculation and analysis of spin phenomena.

In this paper an exact solution of the Dirac equation and the spectrum of particle quasi-energies for fields of the Redmond configuration is found, taking into account the electron anomalous magnetic moment. We consider the case of circular wave polarization using the method suggested in [2]. In addition, electron and positron states are distinguished by introducing the quantum number q_v which is positive and negative for electrons and positrons respectively.

Furthermore, it is shown that by taking into account the electron anomalous magnetic moment the degeneracy in the quantum spin states is removed. Moreover, owing to the presence of the anomalous magnetic moment and at certain values of other parameters, the relative positions of the quasi–energy levels are radically changed. This latter is important for the consideration of spin effects. The establishment of the quasi–energy spectrum allows us to obtain the photon absorption and radiation spectrum. It is found that the radiation frequency is shifted because of the influence of the intensity of the wave field and that this shift can be considerable.

These theoretical considerations are of direct relevance for various applications: laser acceleration [4]; electron–positron pair creation at high wave field intensities [5]; fast measurement of the absolute energy of charged unpolarized particles with a relative precision of 10^{-4} over a wide energy range up to TeV energies, and so on. It is important that during the measurement of the energy of longitudinally spin–polarized electrons by this method, the degree of polarization does not decrease.

2 The solution of the Dirac equation

In this section we derive the solution of the Dirac equation in the field of an electromagnetic wave and a homogeneous magnetic field taking account of the electron anomalous magnetic moment. We describe the external fields in terms of a classical vector-potential

$$A(x) = A^L(x) + A^B(\vec{x}_\perp), \tag{1}$$

where

$$A^B(\vec{x}_\perp) = \Big(0; \ -\ \frac{\mathcal{B}}{2}x^2; \ \frac{\mathcal{B}}{2}x^1; \ 0\Big) \tag{2}$$

is the vector-potential of a constant and homogeneous magnetic field $\vec{\mathcal{B}}$, directed along the z ($\equiv x^3$) axis, [1] and

$$A^L(u) = \Big(0; \ -\ \frac{\mathcal{E}}{\omega}\sin\sqrt{2}\omega u; \ \lambda\frac{\mathcal{E}}{\omega}\cos\sqrt{2}\omega u; \ 0\Big) \tag{3}$$

is the vector potential of the monochromatic circularly polarized wave propagating along $\vec{\mathcal{B}}$ where $u = (x^0 - x^3)/\sqrt{2}$ and $\lambda = \pm 1$ corresponds to the right or left polarized wave. Lastly, ω and $\mathcal{E}$ are the frequency and amplitude of the wave. Note that we choose units for which $\hbar = c = 1$.

We next consider the Dirac equation in the field $A(x)$ taking into account the electron anomalous magnetic moment. This has the form:

$$\Big(\gamma^\mu \hat{\Pi}_\mu - m - ia|\mu_0|F_{\mu\nu}\sigma^{\mu\nu}\Big)\Psi_D(x) = 0, \tag{4}$$

which can be conveniently rewritten as:

$$\Big\{\gamma_u \hat{P}_v + \gamma_v \hat{P}_u - (\vec{\gamma}\hat{\vec{\Pi}}_\perp) - m - a|\mu_0|[\mathcal{B}\Sigma_z + (\vec{\mathcal{H}}\vec{\Sigma})\gamma_u\gamma_v]\Big\}\Psi_D(x) = 0. \tag{5}$$

Here $F_{\mu\nu}$ is the external electromagnetic field tensor, $a = (g-2)/2$ is the gyromagnetic anomaly and $\mu_0 = e/2m$ is the normal electron magnetic moment.

Furthermore we have defined:

$$\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) = (\vec{\alpha}; i\vec{\Sigma}); \quad \vec{\alpha} = \gamma^0\vec{\gamma}; \quad \gamma_u = \frac{\gamma^0 + \gamma^3}{\sqrt{2}}; \quad \gamma_v = \frac{\gamma^0 - \gamma^3}{\sqrt{2}};$$

$$P_u = \frac{P^0 + P^3}{\sqrt{2}}; \quad P_v = \frac{P^0 - P^3}{\sqrt{2}}; \quad v = \frac{x^0 + x^3}{\sqrt{2}},$$

and we use the symbol ' ˆ ' to denote differential operators so that

$$\hat{P}_v = i\frac{\partial}{\partial v} = i\partial_v = \frac{i}{\sqrt{2}}\Big(\frac{\partial}{\partial x^0} + \frac{\partial}{\partial x^3}\Big); \quad \hat{P}_u = i\frac{\partial}{\partial u} = i\partial_u = \frac{i}{\sqrt{2}}\Big(\frac{\partial}{\partial x^0} - \frac{\partial}{\partial x^3}\Big);$$

[1] In this paper we use two notations in parallel according to convenience: $x^1 \equiv x$, $x^2 \equiv y$, $x^3 \equiv z$.

$$\hat{\Pi}_\mu = \hat{P}_\mu + |e|A_\mu(x); \quad \hat{\vec{\Pi}}_\perp = \hat{\vec{P}}_\perp + |e|\vec{A}_\perp; \quad \gamma^\mu \hat{P}_\mu = \gamma_u \hat{P}_v + \gamma_v \hat{P}_u - (\vec{\gamma}\hat{\vec{P}}_\perp).$$

Note that $\gamma_u\gamma_v + \gamma_v\gamma_u = 2$ and $\gamma_v^2 = \gamma_u^2 = 0$.

In equation (5) we have used the relation $\vec{\mathcal{H}} = \vec{n} \wedge \vec{\mathcal{E}}$ for a plane wave, where $\vec{n} = \vec{k}/|\vec{k}|$ and where in turn $\vec{k}$ is the wave vector of the electromagnetic wave. Then $(\vec{\mathcal{H}}\vec{\Sigma})\alpha_3 = i(\vec{\mathcal{E}}\vec{\alpha})$.

Since the expression in the curly brackets in (5) does not depend on the variable v explicitly, we can separate off the v-dependence of the function $\Psi_D(x)$. To do this we introduce the replacement:

$$\Psi_D(x) = e^{-i(vf_v + uf_u)}\Psi_f(u, \vec{x}_\perp), \tag{6}$$

where $f_u = (f^0 + f^3)/\sqrt{2}$ and $f_v = (f^0 - f^3)/\sqrt{2}$ are arbitrary constant numbers whose meaning will emerge later. Then $\Psi_f(u, \vec{x}_\perp)$ satisfies the equation

$$\Big\{\gamma_u f_v + \gamma_v f_u + \gamma_v i\partial_u - (\vec{\gamma}\hat{\vec{\Pi}}_\perp) - m - a|\mu_0|[\mathcal{B}\Sigma_z + (\vec{\mathcal{H}}\vec{\Sigma})\gamma_u\gamma_v]\Big\}\Psi_f(u, \vec{x}_\perp) = 0. \tag{7}$$

Furthermore, following [2] we make the transformation:

$$\Psi_f(u, \vec{x}_\perp) = \Big[1 + \frac{(k\gamma)}{2(kf)}(\vec{\gamma}\hat{\vec{\Pi}}_\perp)\Big]\Phi(u, \vec{x}_\perp) \tag{8}$$

and since, as shown in [2], the function Φ can be chosen to be an eigenfunction of the spin operator Σ_3 (projection of $\vec{\Sigma}$ on to $\vec{\mathcal{B}}$ direction) with eigenvalues $\zeta = \pm 1$:

$$\Sigma_3\Phi = \zeta\Phi, \tag{9}$$

we obtain

$$\begin{aligned}\Big[\gamma_u f_v + \gamma_v f_u + \gamma_v\Big(i\partial_u - \frac{\hat{\vec{\Pi}}_\perp^2}{2f_v} - \frac{|e|\mathcal{B}\zeta}{2f_v}\Big) + \gamma_v(\vec{\gamma}\hat{\vec{\Pi}}_\perp)\frac{a|\mu_0|\mathcal{B}\zeta}{f_v} - \\ -m_* - a|\mu_0|\gamma_v\gamma_u(\vec{\mathcal{H}}\vec{\Sigma})\Big]\Phi(u, \vec{x}_\perp) = 0,\end{aligned} \tag{10}$$

where $m_* = m + a|\mu_0|\zeta\mathcal{B}$.

From (10), after some simple manipulations, we obtain

$$\Big[i\partial_u - \frac{\hat{\vec{\Pi}}_\perp^2}{2f_v} - \frac{|e|\mathcal{B}\zeta}{2f_v} + \frac{f^2 - m_*^2}{2f_v}\Big]\Phi(u, \vec{x}_\perp) = 0, \tag{11}$$

$$\Big[(\gamma q) - m\Big]\Phi(u, \vec{x}_\perp) = 0, \tag{12}$$

where $f^2 = 2f_uf_v$ and where we have introduced a new four-momentum vector q^μ defined by

$$q^\mu = f^\mu\frac{m}{m_*} - \frac{k^\mu}{2(kf)}\frac{m}{m_*}(f^2 - m_*^2) \tag{13}$$

with nonvanishing components $q_v = (q^0 - q^3)/\sqrt{2}$ and $q_u = (q^0 + q^3)/\sqrt{2}$.

It is seen from (11) and (12) that the Φ function is a product of spinor and spatial parts

$$\Phi(u, \vec{x}_\perp) = \chi(u, \vec{x}_\perp) U_{q,\zeta}, \tag{14}$$

where $U_{q,\zeta}$ is a constant bispinor independent of the variables $(u, \vec{x}_\perp)$ satisfying the equations

$$\Big[(\gamma q) - m\Big] U_{q,\zeta} = 0, \tag{15}$$

$$\Sigma_3 U_{q,\zeta} = \zeta U_{q,\zeta}. \tag{16}$$

Because $k^2 = 0$ we have from (13)

$$(kq) = (kf)\frac{m}{m_*}, \quad q_v = f_v \frac{m}{m_*}, \quad q^2 = m^2. \tag{17}$$

So q^μ is a free particle 4–momentum with non–zero components $q_u, \ q_v$.

In (14) $\chi(u, \vec{x}_\perp)$ is a spin independent function satisfying the equation

$$\Big(i\partial_u - \frac{\hat{\vec{\Pi}}_\perp^2}{2f_v} - \frac{|e|\mathcal{B}\zeta}{2f_v} + \frac{f^2 - m_*^2}{2f_v}\Big)\chi(u, \vec{x}_\perp) = 0. \tag{18}$$

This can be written in a more convenient form as:

$$\Big(i\partial_u - \frac{\hat{\vec{\Pi}}_\perp^2}{2f_v}\Big)\chi_0(u, \vec{x}_\perp) = 0 \tag{19}$$

by introducing the replacement

$$\chi(u, \vec{x}_\perp) = \exp\Big[-iu\Big(\frac{|e|\mathcal{B}\zeta}{2f_v} - \frac{f^2 - m_*^2}{2f_v}\Big)\Big]\chi_0(u, \vec{x}_\perp). \tag{20}$$

If we assume that $A(x) \to 0$ for $x \to \infty$, then $\vec{\mathcal{B}}, \vec{\mathcal{E}}, \vec{\mathcal{H}} \to 0, \ \ m_* \to m, \ \ f^\mu \to q^\mu$ and $[(\gamma f) - m]U_{f,\zeta} = 0$. Then for $x \to \infty$ $\Psi_D(x)$ is a solution of the Dirac equation for a free particle, and $f^0, \, f^3$ are then the energy and z component of the electron momentum in the absence of external fields.

In order to separate $\Psi_D(x)$ into electron and positron states we note that

$$q^2 = 2q_u q_v = (q^0)^2 - (q^3)^2 = m^2 \tag{21}$$

so that

$$q^0 = \frac{2q_v^2 + m^2}{2\sqrt{2}q_v}. \tag{22}$$

It is clear from (22) that the signs of q^0 and q_v always coincide, so that $q_v > 0$ (or $f_v > 0$ because of (17)) - corresponds to an electron state and $q_v < 0$ (or $f_v < 0$) corresponds to a positron state [6].

Taking into account that the potential (1) satisfies the condition $\mathrm{div}\vec{A} = 0$, we obtain from (19)

$$i\partial_u \chi_0(u, \vec{x}_\perp) = \frac{1}{2f_v}\Big[\hat{\vec{P}}_\perp^2 + 2|e|(\vec{A}\hat{\vec{P}}_\perp) + e^2\vec{A}_\perp^2\Big]\chi_0(u, \vec{x}_\perp). \tag{23}$$

The solution of equation (23) is simplified if we transform from the coordinate representation into the Fock space (number state) representation by expressing $(x; \hat{P}_x)$, $(y; \hat{P}_y)$ in terms of creation and annihilation operators $(a; a^+)$, $(b; b^+)$ for a harmonic oscillator. Then we find from (23)

$$i\partial_u \chi_0(u, \vec{x}_\perp) = \Big[\frac{\omega_f}{2}(a^+a + \frac{1}{2}) + \frac{\omega_f}{2}(b^+b + \frac{1}{2}) + \frac{\alpha}{\sqrt{2}}(a - ib) +$$
$$+\frac{\alpha^*}{\sqrt{2}}(a^+ + ib^+) + i\frac{\omega_f}{2}(ab^+ - ba^+) + \frac{e^2}{2f_v}\Big(A^L(u)\Big)^2\Big]\chi_0(u, \vec{x}_\perp), \tag{24}$$

where

$$\alpha = \frac{|e|\omega_f}{\sqrt{2m\omega_c}}\Big(A_y^L(u) - iA_x^L(u)\Big) = \frac{\lambda|e|\mathcal{E}\ \omega_f}{\omega\sqrt{2m\omega_c}}\exp(i\sqrt{2}\lambda\omega u),\ \omega_f = \frac{|e|\mathcal{B}}{f_v},\ \omega_c = \frac{|e|\mathcal{B}}{m}. \tag{25}$$

The term $\frac{e^2}{2f_v}\Big(A^L(u)\Big)^2$ in square brackets in (24) can be eliminated by the replacement:

$$\chi_0(u, \vec{x}_\perp) = \chi_1(u, \vec{x}_\perp)\exp\Big[-i\frac{e^2}{2f_v}\int_0^u \Big(A^L(\tau)\Big)^2 d\tau\Big]. \tag{26}$$

By substituting (26) into (24) we then obtain an equation for $\chi_1(u, \vec{x}_\perp)$:

$$i\partial_u \chi_1(u, \vec{x}_\perp) = \Big[\frac{\omega_f}{2}(a^+a + \frac{1}{2}) + \frac{\omega_f}{2}(b^+b + \frac{1}{2}) + \frac{\alpha}{\sqrt{2}}(a - ib) +$$
$$+\frac{\alpha^*}{\sqrt{2}}(a^+ + ib^+) + i\frac{\omega_f}{2}(ab^+ - ba^+)\Big]\chi_1(u, \vec{x}_\perp). \tag{27}$$

We can eliminate the terms linear in a, a^+, b, b^+ in the square brackets in (27) by using the translation operators $D_{\sigma_a} = \exp\Big(\sigma_a a^+ - \sigma_a^* a\Big)$ and $D_{\sigma_b} = \exp\Big(\sigma_b b^+ - \sigma_b^* b\Big)$ which have the following properties:

$$D_{\sigma_a}^{-1} a D_{\sigma_a} = a + \sigma_a,\quad D_{\sigma_a}^{-1} a^+ D_{\sigma_a} = a^+ + \sigma_a^*,\quad D_{\sigma_a}^+ = D_{\sigma_a}^{-1},\quad D_{\sigma_a}^{-1} D_{\sigma_a} = 1.$$

The operator D_{σ_b} has analogous properties. By applying the substitution

$$\chi_1(u, \vec{x}_\perp) = D_{\sigma_a} D_{\sigma_b} \chi_2(u, \vec{x}_\perp) \tag{28}$$

to (27) and using the relation

$$\partial_u D_{\sigma_a} = D_{\sigma_a}\Big\{(\partial_u\sigma_a)a^+ - (\partial_u\sigma_a^*)a - \frac{1}{2}[\sigma_a(\partial_u\sigma_a^*) - \sigma_a^*(\partial_u\sigma_a)]\Big\}$$

and an analogous relation for D_{σ_b} we obtain from (27) the equation for χ_2. By grouping the terms according to their a^+, a, b^+, b content and by setting the coefficients of the terms linear in a^+, a, b^+, b to zero we then obtain the equations:

$$i\partial_u\sigma_a = \frac{\omega_f}{2}\sigma_a + \frac{\alpha^*}{\sqrt{2}} - i\frac{\omega_f}{2}\sigma_b$$

$$i\partial_u\sigma_b = \frac{\omega_f}{2}\sigma_b + i\frac{\alpha^*}{\sqrt{2}} + i\frac{\omega_f}{2}\sigma_a$$

From these we find that the parameters σ_a, σ_b obey the condition $i\sigma_a - \sigma_b = const.$ If we now choose $i\sigma_a = \sigma_b$ then for χ_2 and σ_a we obtain the equations:

$$i\partial_u\chi_2(u,\vec{x}_\perp) = \Big[\frac{\omega_f}{2}(a^+a + \frac{1}{2}) + \frac{\omega_f}{2}(b^+b + \frac{1}{2}) + \\ + \frac{1}{\sqrt{2}}(\alpha\sigma_a + \alpha^*\sigma_a^*) + i\frac{\omega_f}{2}(ab^+ - ba^+)\Big]\chi_2(u,\vec{x}_\perp), \tag{29}$$

$$\partial_u\sigma_a + i\omega_f\sigma_a = -i\frac{\alpha^*}{\sqrt{2}}. \tag{30}$$

We can eliminate the term $\frac{1}{\sqrt{2}}(\alpha\sigma_a + \alpha^*\sigma_a^*)$ in (29) by the replacement

$$\chi_2(u,\vec{x}_\perp) = \exp\Big[-i\sqrt{2}\int_0^u Re\Big(\alpha(\tau)\sigma(\tau)\Big)d\tau\Big]\chi_3(u,\vec{x}_\perp) \tag{31}$$

and by substituting (31) in (29) we obtain an equation for $\chi_3(u,\vec{x}_\perp)$:

$$i\partial_u\chi_3(u,\vec{x}_\perp) = \Big[\frac{\omega_f}{2}(a^+a + \frac{1}{2}) + \frac{\omega_f}{2}(b^+b + \frac{1}{2}) + i\frac{\omega_f}{2}(ab^+ - ba^+)\Big]\chi_3(u,\vec{x}_\perp). \tag{32}$$

If now in (32) we return to a coordinate representation by introducing cylindrical coordinates (ϱ,φ), we find

$$i\partial_u\chi_3(u,\vec{x}_\perp) = \Big[\frac{\omega_f^2 f_v}{8}\varrho^2 - \frac{1}{2f_v}\Big(\frac{\partial^2}{\partial\varrho^2} + \frac{1}{\varrho}\frac{\partial}{\partial\varrho} + \frac{1}{\varrho^2}\frac{\partial^2}{\partial\varphi^2}\Big) - i\frac{\omega_f}{2}\frac{\partial}{\partial\varphi}\Big]\chi_3(u,\vec{x}_\perp). \tag{33}$$

Since the expression in the square bracket in (33) does not depend on the variable u we can factor $\chi_3(u,\vec{x}_\perp)$ into the form

$$\chi_3(u,\vec{x}_\perp) = \exp(-iE_n u)\phi_{n,s}(\varrho,\varphi), \tag{34}$$

where $\phi_{n,s}(\varrho,\varphi)$ obeys the equation

$$\Big[\frac{\omega_f^2 f_v}{8}\varrho^2 - \frac{1}{2f_v}\Big(\frac{\partial^2}{\partial\varrho^2} + \frac{1}{\varrho}\frac{\partial}{\partial\varrho} + \frac{1}{\varrho^2}\frac{\partial^2}{\partial\varphi^2}\Big) - i\frac{\omega_f}{2}\frac{\partial}{\partial\varphi} - E_n\Big]\phi_{n,s}(\varrho,\varphi) = 0 \tag{35}$$

and the parameter E_n will be defined below. Furthermore, we can factor $\phi_{n,s}(\varrho,\varphi)$ into terms depending on ϱ and φ:

$$\phi_{n,s}(\varrho,\varphi) = \exp(il\varphi)R(\varrho). \tag{36}$$

By introducing the dimensionless variable $\eta = \frac{|e|\mathcal{B}}{2}\varrho^2$ the equation for R can be written in the form

$$\eta R'' + R' + \Big(\varepsilon - \frac{\eta}{4} - \frac{l^2}{4\eta}\Big)R = 0, \tag{37}$$

where $\varepsilon = \frac{E_n}{\omega_f} - \frac{l}{2}$ and the prime denotes differentiation with respect to η. Equation (37) has the same form as the equation for the radial wave function of the Schrodinger equation in a homogeneous magnetic field $\vec{\mathcal{B}}$ for states where the electron possesses definite momentum and angular momentum values along the direction of the field $\vec{\mathcal{B}}$ [7]. The solution of the equation (37) can be expressed in terms of Laguerre functions [7], [8]:

$$R(\eta) = N_{n,s}I_{n,s}(\eta), \tag{38}$$

where

$$I_{n,s}(\eta) = \frac{1}{\sqrt{n!s!}}e^{-\frac{\eta}{2}}\eta^{\frac{|l|}{2}}L_s^{|l|}(\eta),$$

$N_{n,s}$ is a normalization factor defined below and $L_s^{|l|}$ are associated Laguerre polynomials. The constraint that the function R must vanish as $\eta \to \infty$ means that $\varepsilon - \frac{|l|+1}{2} = s$, where $s = 0,1,2,...$ is a radial quantum number. This in turn allows us to obtain the values:

$$E_n = \omega_f\Big(s + \frac{l+|l|}{2} + \frac{1}{2}\Big) = \frac{|e|\mathcal{B}}{f_v}\Big(n + \frac{1}{2}\Big) \tag{39}$$

for E_n. Here $n = s + \frac{l}{2} + \frac{|l|}{2} = 0,1,2,...$ is the principle quantum number and l is the value of the component of angular momentum along $\vec{\mathcal{B}}$ (for $l > 0$ we have $l = n-s$; for $l < 0$ we have $n - s = 0$).

Substituting (26) into (20) and using the relations (28), (31), (34), (36), (38), (39) we now obtain the solution of equation (18), namely:

$$\chi(u,\vec{x}_\perp) = \exp\Big\{-i\frac{u}{2f_v}\Big[|e|\mathcal{B}(2n+1+\zeta) + m_*^2 - m^2\Big] -$$
$$-i\frac{e^2}{2f_v}\int\limits_0^u\Big(A^L(\tau)\Big)^2 d\tau - i\sqrt{2}\int\limits_0^u Re\Big(\alpha(\tau)\sigma_a(\tau)\Big)d\tau\Big\}F_{n,s}(u,\vec{x}_\perp), \tag{40}$$

where

$$F_{n,s}(u,\vec{x}_\perp) = D_{\sigma_a} D_{\sigma_b} \phi_{n,s}(x,y), \quad \phi_{n,s}(x,y) = N_{n,s} e^{il\varphi} I_{n,s}(\eta). \tag{41}$$

We are mainly interested in the nonresonant case $((kf) - \lambda|e|\mathcal{B} \neq 0))$ and then the parameter σ_a, found from equation (30) has the form

$$\sigma_a = \frac{|e|\mathcal{B}}{2\sqrt{m\omega_c}} \frac{|e|\mathcal{E}}{\omega} \frac{\exp(-i\sqrt{2}\lambda\omega u)}{(kf) - \lambda|e|\mathcal{B}}. \tag{42}$$

Exactly at the resonance $(kf) - |e|\mathcal{B} = 0$ we have $Re(\alpha\sigma_a) = 0$. Expressing the operators D_{σ_a} and D_{σ_b} in terms of $(x;\hat{P}_x)$, $(y;\hat{P}_y)$ and using the expressions (25), (42) we obtain

$$F_{n,s}(u,\vec{x}_\perp) = e^{i\alpha_1(-x(\sin\sqrt{2}\lambda\omega u)+y(\cos\sqrt{2}\lambda\omega u))} e^{-i\beta_1((\cos\sqrt{2}\lambda\omega u)\hat{P}_x + (\sin\sqrt{2}\lambda\omega u)\hat{P}_y)} \phi_{n,s}(x,y) =$$
$$= \exp\Big[i\alpha_1(-x\sin\sqrt{2}\lambda\omega u + y\cos\sqrt{2}\lambda\omega u)\Big]\phi_{n,s}(x - \beta_1\cos\sqrt{2}\lambda\omega u;\ y - \beta_1\sin\sqrt{2}\lambda\omega u) \tag{43}$$

where

$$\beta_1 = \frac{|e|\mathcal{E}}{\omega}\frac{1}{(kf) - \lambda|e|\mathcal{B}}\ , \quad \alpha_1 = \frac{|e|\mathcal{B}}{2}\beta_1. \tag{44}$$

To calculate the operators D_{σ_a}, D_{σ_b} we used the Weyl identity

$$e^{\hat{A}+\hat{B}} = e^{-\frac{1}{2}[\hat{A},\hat{B}]} e^{\hat{A}} e^{\hat{B}},$$

which is valid for operators $\hat{A}$, $\hat{B}$ satisfying the equality $[\hat{A},[\hat{A},\hat{B}]] = [\hat{B},[\hat{A},\hat{B}]] = 0$.

So from (6),(8),(14),(40) and using (3),(25),(42) we finally obtain the following expression for the electron wave function

$$\Psi_D(x) = e^{-i(Qx)}\Big[1 + \frac{(k\gamma)}{2(kf)}(\vec{\gamma}\hat{\vec{\Pi}}_\perp)\Big] U_{q,\zeta} F_{n,s}(u,\vec{x}_\perp). \tag{45}$$

where

$$Q^\mu = q^\mu \frac{m_*}{m} + \frac{k^\mu}{2(kQ)}\Big[|e|\mathcal{B}(2n+1+\zeta) + \frac{\xi^2 m^2 (kQ)}{(kQ) - \lambda|e|\mathcal{B}}\Big], \tag{46}$$

and where $\xi = |e|\mathcal{E}/m\omega$ is the intensity parameter of the electromagnetic wave field. The wave function (45) satisfies the periodicity conditions

$$\Psi_D(t+T) = e^{-iQ^0 T}\Psi_D(t)\ , \quad \Psi_D(z+\lambda_w) = e^{-iQ^3\lambda_w}\Psi_D(z),$$

where $T = \frac{2\pi}{\omega}$ is the period of the oscillations and λ_w is the wavelength. Therefore according to the definition of quasi-momentum [9],[10], Q^μ is a particle quasi–momentum with components Q^0 and Q^3.

The normalization factor $N_{n,s}$ is obtained from the condition

$$\int \Psi_D^*(x)\Psi_D(x)d^3x = 1. \tag{47}$$

Noticing that

$$\Psi_D^* = \bar{\Psi}\gamma^0, \quad \bar{\Psi}_D = e^{i(Qx)}\bar{U}_{q,\zeta}\Big[1 - \frac{\gamma_v}{2f_v}(\vec{\gamma}\hat{\vec{\Pi}}^*_\perp)\Big]F^*_{n,s}(u,\vec{x}_\perp) \tag{48}$$

and substituting (45), (48) into (47) we obtain

$$\begin{aligned}(\bar{U}_q\gamma^0U_q)\int F^*_{n,s}F_{n,s}d^3x - \frac{1}{2f_v}(\bar{U}_q\gamma_v\gamma^\alpha\gamma^0U_q)\int(\hat{\Pi}^{*\alpha}F^*_{n,s})F_{n,s}d^3x + \\ +\frac{1}{2f_v}(\bar{U}_q\gamma^0\gamma_v\gamma^\alpha U_q)\int F^*_{n,s}(\hat{\Pi}^\alpha F_{n,s})d^3x - \\ -\frac{1}{(2f_v)^2}(\bar{U}_q\gamma_v\gamma^\alpha\gamma^0\gamma_v\gamma^\beta U_q)\int(\hat{\Pi}^{*\alpha}F^*_{n,s})(\hat{\Pi}^\beta F_{n,s})d^3x = 1. \\ \alpha,\beta = 1,2.\end{aligned} \tag{49}$$

Assuming the normalization $\bar{U}_qU_q = 2m$ from (15) we find

$$\bar{U}_q\gamma^\mu U_q = 2q^\mu, \qquad\qquad \mu = 0,3. \tag{50}$$

By using the expressions (41), (43), (50) and taking the formula [8]

$$\int_0^\infty I^2_{n,s}(\eta)d\eta = 1$$

into account we find that the first term of the left part of the expression (49) is given by

$$2q^0\int F^*_{n,s}F_{n,s}d^3x = 2q^0\frac{N^2_{n,s}}{|e|\mathcal{B}}\int_{-L_z/2}^{+L_z/2}dz\int_0^{2\pi}d\varphi\int_0^\infty I^2_{n,s}(\eta)d\eta = 4\pi q^0L_z\frac{N^2_{n,s}}{|e|\mathcal{B}}. \tag{51}$$

If we transform the second integral in (49) using the relation

$$\int(\hat{\Pi}^{*\alpha}F^*_{n,s})F_{n,s}d^3x = \int F^*_{n,s}(\hat{\Pi}^\alpha F_{n,s})d^3x,$$

and then the relation

$$\begin{aligned}-\bar{U}_q\gamma_v\gamma^\alpha\gamma^0U_q + \bar{U}_q\gamma^0\gamma_v\gamma^\alpha U_q = \frac{1}{\sqrt{2}}\bar{U}_q\Big[\gamma_v(\gamma_u+\gamma_v)\gamma^\alpha + (\gamma_u+\gamma_v)\gamma_v\gamma^\alpha\Big]U_q = \\ = \frac{1}{\sqrt{2}}\bar{U}_q(\gamma_v\gamma_u+\gamma_u\gamma_v)\gamma^\alpha U_q = \sqrt{2}\bar{U}_q\gamma^\alpha U_q = 0\end{aligned}$$

we see that the second and the third terms of the left hand side of the equality (49) are cancelled. Noticing that

$$\bar{U}_q\gamma_v\gamma^\alpha\gamma^0\gamma_v\gamma^\beta U_q = -\frac{1}{\sqrt{2}}\bar{U}_q\gamma^\alpha\gamma_v(\gamma_u+\gamma_v)\gamma_v\gamma^\beta U_q = -\sqrt{2}\bar{U}_q\gamma^\alpha\gamma_v\gamma^\beta U_q = \\ = \sqrt{2}\bar{U}_q\gamma_v\gamma^\alpha\gamma^\beta U_q$$

and transforming the fourth integral in (49) by using the relation

$$\int(\hat{\Pi}^{*\alpha}F^*_{n,s})(\hat{\Pi}^\beta F_{n,s})d^3x = \int F^*_{n,s}(\hat{\Pi}^\alpha\hat{\Pi}^\beta F_{n,s})d^3x$$

we can write the last term on the left hand side of (49) in the form

$$\begin{aligned} &-\frac{\sqrt{2}}{(2f_v)^2}(\bar{U}_q\gamma_v\gamma^\alpha\gamma^\beta U_q)\int F^*_{n,s}(\hat{\Pi}^\alpha\hat{\Pi}^\beta F_{n,s})d^3x = \\ &= \frac{\sqrt{2}}{(2f_v)^2}(\bar{U}_q\gamma_v U_q)\int F^*_{n,s}\big[\hat{\vec{\Pi}}_\perp^2 + i\zeta(\hat{\Pi}^1\hat{\Pi}^2 - \hat{\Pi}^2\hat{\Pi}^1)\big]F_{n,s}d^3x = \\ &= \frac{\sqrt{2}\ 2\pi L_z}{(2f_v)^2}\frac{N^2_{n,s}}{|e|\mathcal{B}}2q_v|e|\mathcal{B}\zeta + \frac{\sqrt{2}}{(2f_v)^2}2q_v\int F^*_{n,s}(\hat{\vec{\Pi}}_\perp^2 F_{n,s})d^3x. \end{aligned} \tag{52}$$

Here we used the relations (51), the relations $\hat{\Pi}^1\hat{\Pi}^2 - \hat{\Pi}^2\hat{\Pi}^1 = -i|e|\mathcal{B}$, $\Sigma_3 U_q = i\gamma^1\gamma^2 U_q = \zeta U_q$ and $\bar{U}_q\gamma_v U_q = 2q_v$ (following from (15) and (50)).

From (18), (40) we obtain the relation

$$\Big[|e|\mathcal{B}(2n+1) + \frac{m^2\xi^2(kf)}{(kf)-\lambda|e|\mathcal{B}} + 2if_v\partial_u\Big]F_{n,s}(u,\vec{x}_\perp) = \hat{\vec{\Pi}}_\perp^2 F_{n,s}(u,\vec{x}_\perp). \tag{53}$$

Substituting $\hat{\vec{\Pi}}_\perp^2 F_{n,s}$ from (53) into (52) and using (43), (51) we find

$$\begin{aligned} &\int F^*_{n,s}(\hat{\vec{\Pi}}_\perp^2 F_{n,s})d^3x = \Big[|e|\mathcal{B}(2n+1) + \frac{m^2\xi^2(kf)}{(kf)-\lambda|e|\mathcal{B}}\Big]\int F^*_{n,s}F_{n,s}d^3x + \\ &+i\int F^*_{n,s}(\partial_u F_{n,s})d^3x = 2\pi L_z\frac{N^2_{n,s}}{|e|\mathcal{B}}\Big[|e|\mathcal{B}(2n+1) + \frac{m^2\xi^2(kf)^2}{((kf)-\lambda|e|\mathcal{B})^2}\Big]. \end{aligned} \tag{54}$$

Substituting (51), (52), (54) into (49) we obtain

$$N_{n,s} = \Big(\frac{|e|\mathcal{B}}{4\pi L_z q^0}\Big)^{\frac{1}{2}}\Big\{1 + \frac{(q^0+q^3)}{2q^0}\Big[\frac{|e|\mathcal{B}}{m^2_*}(2n+1+\zeta) + \frac{\xi^2(kq)^2}{((kq)\frac{m_*}{m} - \lambda|e|\mathcal{B})^2}\Big]\Big\}^{-\frac{1}{2}}. \tag{55}$$

3 The electron and positron quasi–energy spectra

In the approximation that $m^2_* \simeq m^2 + |e|\mathcal{B}\zeta a$ (i.e. neglecting the term $(|e|\mathcal{B}a/2m)^2$ because it is small) from (46) we obtain a dispersion equation

$$Q^2 = (Q^0)^2 - (Q^3)^2 = m^2 + |e|\mathcal{B}(2n+1+\zeta+\zeta a) + \frac{m^2\xi^2(kQ)}{(kQ)-\lambda|e|\mathcal{B}}. \tag{56}$$

In particular, as $\vec{\mathcal{B}} \to 0$ from (46) we obtain the known [3] value of the 4-quasi-momentum (the average over time of the kinetic 4-momentum) of an electron in a plane wave electromagnetic field:

$$Q^\mu = q^\mu + \frac{m^2\xi^2}{2(kq)}k^\mu. \tag{57}$$

For the case when $\xi \to 0$, (56) gives us the usual expression for the energy of an electron with anomalous magnetic moment in a constant magnetic field (in the approximation $\mathcal{B} \ll \mathcal{B}_c = m^2/|e|$)[11]:

$$(Q^0)^2 = (Q^3)^2 + m^2 + |e|\mathcal{B}(2n+1+\zeta+\zeta a). \tag{58}$$

Let us now find the relative positions of the quasi–energy levels $Q^0_{n,\zeta}$ for a circularly polarized wave. Due to the condition $k^2 = 0$, from (46) we have

$$(kQ) = (kq)\Big(1 + \zeta\frac{a\omega_c}{2m}\Big) \tag{59}$$

and the following expressions for the quasi–energy $Q^0_{n,\zeta}$ and quasi–momentum $Q^3_{n,\zeta}$:

$$Q^0_{n,\zeta} = q^0\Big(1+\zeta G\Big) + \frac{\gamma\omega_c(2n+1+\zeta)}{(1+\zeta G)} + \frac{\gamma m\xi^2}{1-\lambda\frac{2\gamma\omega_c}{\omega}+\zeta G}, \tag{60}$$

$$Q^3_{n,\zeta} = q^3(1+\zeta G) + \frac{\gamma\omega_c(2n+1+\zeta)}{(1+\zeta G)} + \frac{\gamma m\xi^2}{1-\lambda\frac{2\gamma\omega_c}{\omega}+\zeta G}\ , \tag{61}$$

where for brevity we use $G = a\omega_c/2m$ and $\gamma = (q^0+q^3)/2m$.

Hence, we can see that $Q^3_{n,\zeta}$ depends on the quantum numbers n and ζ, and the difference between the quasi–energy levels with similar ζ is

$$Q^0_{n,\zeta} - Q^0_{n',\zeta} = Q^3_{n,\zeta} - Q^3_{n',\zeta} = \frac{2\gamma\omega_c(n-n')}{(1+\zeta G)},$$

i.e. the transition $Q^0_{n,\zeta} \to Q^0_{n',\zeta}$ is associated with the transition between the states with different values of the z component of the quasi–momentum. Since G is usually $\ll 1$ (near a resonance we have: $G = \frac{a}{4}\frac{\hbar\omega}{mc^2}\frac{1}{\gamma} \approx \frac{10^{-9}}{4}\frac{1}{\gamma} \ll 1$) then in the approximation that we neglect G the difference between the neighbouring quasi-energy levels with similar ζ is approximately

$$Q^0_{n+1,\zeta} - Q^0_{n,\zeta} \simeq 2\gamma\omega_c. \tag{62}$$

We will now find the relative positions of quasi–energy levels corresponding to opposite spin directions. If

$$G \ll |\delta| = |1 - \lambda\frac{2\gamma\omega_c}{\omega}|$$

from (60),(61) we obtain the approximate relationship:

$$Q^0_{0,+1} - Q^0_{0,-1} = Q^3_{0,+1} - Q^3_{0,-1} = 2\gamma\omega_c(1 + \frac{a}{2} - a\frac{\xi^2}{\delta^2}). \tag{63}$$

The quantity δ describes the detuning of the frequencies ω and $2\gamma\omega_c$. In the case of weak wave fields i.e. when $\xi^2/\delta^2 \ll 1$, (63) gives us $Q^0_{0,+1} - Q_{0,-1} = 2\gamma\omega_c(1 + \frac{a}{2})$. Using this relation and (62) we find the quasi–energy spectrum shown schematically in Fig 1. This shows that inclusion of the electron anomalous magnetic moment removes the degeneracy between the levels $Q^0_{n,+1}$ and $Q^0_{n+1,-1}$.

For intense wave fields, i.e. when $a\xi^2/\delta^2 \gg 1$, from (63) it follows that the relative positions of the quasi–energy levels with opposite spin direction change radically (Fig.2). On some levels in the lower part of the spectrum the electrons can have only one spin direction and the number of these levels is $(Q^0_{0,-1} - Q^0_{0,+1})/2\gamma\omega_c)$.

From these considerations of the quasi–energy spectrum it is seen that inclusion of the anomalous magnetic moment in the electron quasi–energy spectrum is essential at all values of ξ^2/δ^2.

Note that by summing the values of Q^μ found for $\lambda = +1$ and $\lambda = -1$ from (46) we find the following expression for the particle quasi-energy in linearly polarized waves:

$$Q^\mu = q^\mu\frac{m_*}{m} + \frac{k^\mu}{2(kQ)}\Big[|e|\mathcal{B}(2n + 1 + \zeta) + \frac{m^2\xi^2(kQ)^2}{(kQ)^2 - (e\mathcal{B})^2}\Big].$$

Hence, in the approximation that $m_*^2 \simeq m^2 + |e|\mathcal{B}\zeta a$, we obtain the dispersion equation

$$Q^2 = (Q^0)^2 - (Q^3)^2 = m^2 + |e|\mathcal{B}(2n + 1 + \zeta + \zeta a) + \frac{m^2\xi^2(kQ)^2}{(kQ)^2 - (e\mathcal{B})^2}$$

for a linearly polarized wave (taking account of the electron anomalous magnetic moment) which agrees with the analogous expression (except for the absent anomalous magnetic moment) in [5].

4 The photon absorption and radiation spectrum

For absorption of a photon with 4–momentum $k^\mu(\omega;\vec{k})$ by an electron moving at an angle $\theta = 0$ to the z–axis, the energy–momentum conservation law reads as:

$$Q^\mu + k^\mu = Q'^\mu. \tag{64}$$

From this and using Q^μ from (46) we find that the photon absorption frequency ω is:

$$\omega = \frac{\gamma\omega_c[2(n' - n) + (\zeta' - \zeta)(1 + a)]}{1 + \zeta G}. \tag{65}$$

For $G = 0$ and without spin flip the frequency ω coincides with that found earlier by classical methods.

We see from (65) that if there is no spin flip, the frequencies of the Q^0 spectrum are equally spaced. Then the electron can resonantly absorb a succession of photons of the same frequency and accelerate.

For radiation of a photon with 4–momentum $k'(\omega';\vec{k}')$ by an electron the energy–momentum conservation equation is given by:

$$Q^\mu + \nu k^\mu = Q'^\mu + k'^\mu \qquad (66)$$

where the coefficient ν denotes the number of photons absorbed from the wave field.

From (66) and using Q^μ from (46) we find a cubic equation for the frequency spectrum ω' namely:

$$g_3\omega'^3 - g_2\omega'^2 + g_1\omega' - g_0 = 0\ , \qquad (67)$$

where

$$g_0 \equiv \frac{\nu\omega}{\gamma} + \omega_c[2(n-n') + (\zeta-\zeta')(1+a)]\ ,$$

$$g_1 \equiv \frac{(1+\cos\theta)(1+\zeta G)}{2\gamma} + 2\gamma(1-\cos\theta)\Big\{1+\zeta G + \frac{\omega_c}{m}\frac{(2n+1+\zeta)}{1+\zeta G} + \frac{\omega_c}{m}\frac{[2(n-n')+(\zeta-\zeta')(1+a)]}{\delta+\zeta G} + \frac{\xi^2(1+\zeta G)}{(\delta+\zeta G)^2} + \frac{\nu\omega}{m\gamma}(1+\frac{1+\zeta G}{\delta+\zeta G})\Big\}\ ,$$

$$g_2 \equiv \frac{\sin^2\theta}{m} + \frac{(1-\cos\theta)}{m(\delta+\zeta G)}\Big\{4\gamma^2(1-\cos\theta)[1+\zeta G + \frac{\omega_c}{m}\frac{(2n+1+\zeta)}{1+\zeta G} + \frac{\xi^2}{\delta+\zeta G}] + (1+\cos\theta)(1+\zeta G) + \frac{4\gamma\nu\omega}{m}(1-\cos\theta)\Big\}\ ,$$

$$g_3 \equiv \frac{2\gamma\sin^2\theta(1-\cos\theta)}{(\delta+\zeta G)m^2}\ .$$

For photons radiated along the z–axis ($\cos\theta = 1$) we find from (67) that:

$$\omega' = \frac{g_0}{g_1} = \frac{\nu\omega + \gamma\omega_c[2(n-n')+(\zeta-\zeta')(1+a)]}{1+\zeta G}\ . \qquad (68)$$

For the radiation of photons at optical and lower frequencies the quantum corrections in (67) can be neglected and then, for an arbitrary θ, we obtain:

$$\omega' = \frac{\nu\omega + 2\gamma\omega_c(n-n')}{\frac{1+\cos\theta}{2} + 2\gamma^2(1-\cos\theta)(1+\frac{\xi^2}{\delta^2})}\ . \qquad (69)$$

We see from (69) that when $\cos\theta \neq 1$ the term ξ^2/δ^2 would lead to a considerable change, namely a decrease, in the radiation frequency ω' compared with the scattering of an electromagnetic wave on free electrons — especially near resonance when $\delta \ll 1$.

Because the intensity of the radiation increases near resonance like ξ^2/δ^2 [12], the resonance condition $\omega = 2\gamma\omega_c$ can be used for measurement of the particle energy and then the shift of the radiation frequency ω' (69) compared to the incident laser frequency ω allows incident and radiated photons to be distinguished [13].

Note that the resonance condition $\omega = 2\gamma\omega_c$ is easy to realise using existing powerful lasers (with ξ up to 1) for reasonable values of the parameters γ and ω_c. For example at an electron energy of 500 GeV and a laser wavelength $\lambda_w = 0.248\mu m$ ($\hbar\omega = 5eV$) from a KrF laser, one needs a field $\mathcal{B} = 220\ Gs$.

5 Summary

We have found the wave functions (45) and electron quasi-energy spectrum (46) and (56) in a superposition of a homogeneous magnetic field $\vec{\mathcal{B}}$ and a classical circularly polarized wave propagating along $\vec{\mathcal{B}}$ for an electron with anomalous magnetic moment. The factorized wave functions (45) facilitate calculations for processes involving particles with spin since the bispinor $U_{q,\zeta}$ satisfies the equation for a free particle.

Taking account of the electron anomalous magnetic moment removes the degeneracy of the quantum spin states. For intense wave fields, i.e. when $a\xi^2/\delta^2 \gg 1$ the relative positions of the quasi–energy levels with opposite spin direction change radically (Fig. 2).

We have considered the photon absorption and radiation spectrum and have found a dependence of the shift of the radiation frequency on the intensity of the wave field.

In addition we suggest a new method for measuring the electron energy by observing the radiation intensity near resonance. This will be discussed in more detail in a later paper. This method could also be used for ultra high energy muons [14].

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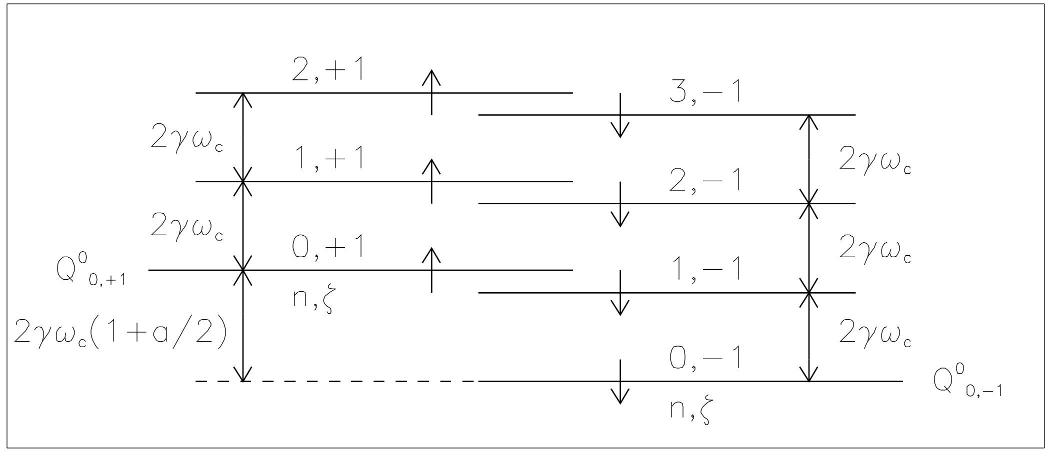
Figure captions

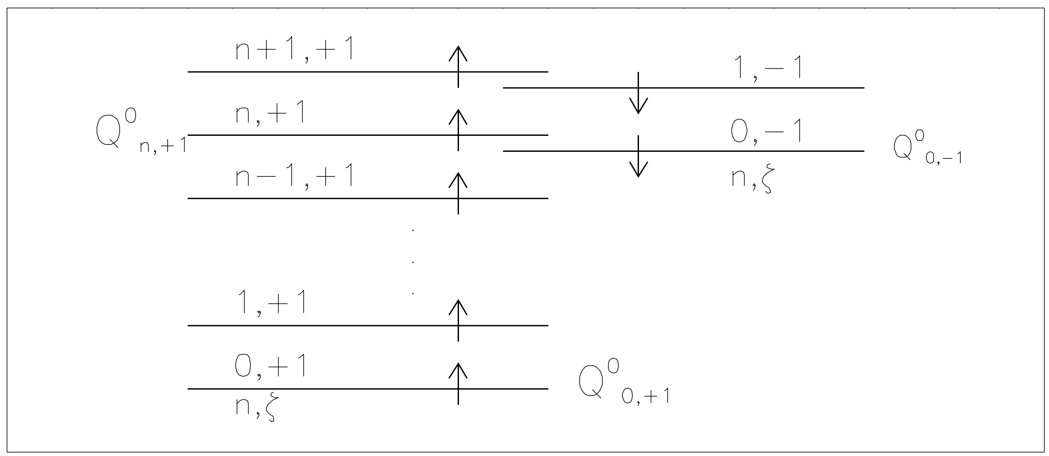
1.Sketch of the electron quasi–energy spectrum for $\xi^2/\delta^2 \ll 1$.

2.Sketch of the electron quasi–energy spectrum for $a\xi^2/\delta^2 \gg 1$.



2,+1
3,−1
2γω_c
1,+1
2,−1
2γω_c
0,+1
1,−1
Q^0_{0,+1}
n,ζ
2γω_c(1+a/2)
0,−1
n,ζ
Q^0_{0,−1}

Fig.1



n+1,+1
1,−1
Q^0_{n,+1}
n,+1
0,−1
n,ζ
Q^0_{0,−1}
n−1,+1
1,+1
0,+1
n,ζ
Q^0_{0,+1}

Fig.2